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Lenses formed under impact loading: shape and Poisson's ratio relationship

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When a material is impact-loaded uniformly over a circular area, a lens is formed in the bulk under the contact surface. The interface between the lens and the rest of the body is either a fracture surface or a region of transition from a ductile behaviour inside the lens to a brittle behaviour outside of it. The ductility within the lens is shown to be concomitant with a densification of the lens's material. A linear relationship correlating the angle between the loaded surface and the lens-body interface, α, with Poisson's ratio aquared, μ^2, is shown to exist. This relationship is separable into two linear regions, below snd above $\alpha \sim 150°$, for "ductile" and "brittle" materials.

1. Introduction

Cone fractures in inorganic brittle glasses have been produced by Roesler [1], Benbow [2] and Frank and Lawn [3], in order to test the validity of Griffith's fracture theory. Marsh [4] showed, however, that a significant amount of plastic flow exists in such glasses at room temperature, leading to the formation of spherically symmetrical lens-like region of plastic flow under and around the tip of the indenting device. Similar observations in polymethylmethacrylate (PMMA) were recently reported by Puttick [5]. In all these works, an isotropic response of the tested substances was implicit. As the conical cracks were produced in very similar substances, no general relationship between the crack angle and other bulk properties was deduced.

In this work a wide selection of materials was used, covering the range of Poisson's ratio $0.00 \leqslant \mu \lesssim 0.50$. It will be shown that a relationship between the angle of crack or lens-shoulders and μ^2 exists. This relationship is separable into two linear regions.

2. Experimental

Commercial rod stock of polymers was obtained from Cadillac Plastics and Chemical Company. Moldable epoxy was obtained from Buehler Co and samples were prepared according to the directions of the supplier. The polyoxymethylene (POM) was du Pont's Delrin, the dimethylphenylene oxide (PPO) was General Electric Co, Noryl and the polyvinylchoride (PVC) was General Tire's Boltaron HI-HT4100 CPVC type IV. The polyurethane (PU) rubber was Conap Company's EN4 resin cured for 10 h at 180°F to a Shore A durometer hardness of 93. The natural rubber (NR) was compounded with carbon black and had a Shore A hardness of 70.

The samples were cylindrical in shape with 2 to 2½ in. diameters and lengths ranging from 1 in. up to 3 in. They were impacted, by an indenter, axially at the centre of one of their flat faces. Brittle samples were prevented from shattering by being constrained in sections of heavy metal tubes. The samples were placed in a controlled-temperature sample-holder assembly which was mounted in front of the backstop of an incline impact tester of the Gaynes style. The indenter, a right cylinder ½ in. in diameter with a planar contact area of about 1.1 cm^2, was actuated by being impacted by a 1 ton dolly. The dolly was arrested after recoil to avoid further impacts. The velocity of impact was kept in the range of 45 to 80 cm sec^{-1}, corresponding to 2.8 to 5 kbar pressure on the contact area, with almost all runs being performed at 45 cm sec^{-1}.

In addition to the polymeric materials, samples of cork and glazed concrete tiles were also tested. After impact, most samples were sectioned in half and the cut surfaces polished. Then, measurements of the angles, α, between the loaded surface and the shoulders of the resultant lenses were performed on the samples themselves or on photographs of the polished surfaces. Several typical photographs will be presented herein.

Density measurements were performed in density gradient columns using a water and methanol mixture for polypropylene, and NaCl aqueous solution for PMMA. In both instances less than 0.1% Triton N-101 surfactant (Rohm and Haas Co) was added to the liquids to eliminate air bubbles on the polymer surfaces. The index of refraction, n, of PMMA was determined by the use of calibrated index of refraction liquids in contact with the samples on the monochromatic polarized light microscope stage.

3. Results

The experimental impact conditions yield a uniformly distributed contact stress. The indenter penetrated the various samples to different depths, but in all instances lenses were formed in the volumes extending in front of the deepest penetration of the indenter. When the tip of the indenter is not flat but hemispherical, the area of contact between the tip and the sample changes with time. With the change in area a change in the stress distribution takes place, the point of highest compressive stress being a function of the radius of the contact area [6]. It was found that replacement of the flat-faced indenter by a hemispherical one generates, in PMMA at 25°C, poorly defined lenses that show internal unevenness, indicating variations in the strains under the applied stress. Variations in the applied stress, and resultant strains, are mentioned by Frank and Lawn [3] but are discarded by them as unimportant. We believe that the unevenness caused by the hemispherical indenter is significant and is probably a contributor to the deviation of the quartz datum [7], to be shown later, from the other data points.

The complete data are presented in Table I. In it the α angles of elastomers, non-elastomers, cork and concrete tile, determined by us, are presented together with some α angles obtained from the literature for silicate glass, silica and the 0001 face of α-quartz. In the case of quartz the angle was measured from the photographs of Hartley and Wilshaw [7] and an error of several degrees is possible. Poisson's ratios from the literature are given for all the tabulated substances.

Figs. 1 to 4 show several of the lenses generated with the flat-faced indenter. Fig. 1 is for PMMA. PS yielded practically the same shape lens, except for a larger α angle. Fig. 2 is for PVC and is very similar to PPO. Fig. 3 is for POM, yielding lenses very close to nylon 66 and PP, and Fig. 4 depicts PU, which yields almost the same shape lenses as NR. The lens of PU was dislodged from its position during sectioning so that it is the dislodged and tilted half lens that is shown in Fig. 4. The shape of cracks developed in inorganic glasses is shown in the works of Roesler [1] and Benbow [2]. These

TABLE I Angle, α, and Poisson's ratio, μ, for the investigated materials

No. and material	Abbreviation	Angle α (degrees)	Poisson's ratio, μ
1. Natural rubber	NR	9.0	0.50–0.49 [8, 9]
2. Polyurethane elastomer	PU	26.5	0.45 [10]
3. Polyvinylchloride	PVC	70.0	0.398 [11]
4. Polyoxymethylene	POM	71.0	0.409 [12]
5. Polypropylene	PP	71.5	0.375 [13]
6. Nylon 66	Nylon 66	74.0	0.38 [14]
7. Polytetrafluoroethylene	PTFE	74 ± 1	0.39 [15]
8. Polycarbonate of bisphenol A	PC	84.0	0.38 [16, 17]
9. Epoxy resin, epon 828 base	Epoxy	100.0	0.347 [18]
10. Poly(2,6-dimethyl-1,4-phenylene oxide)	PPO	107.0	0.35 [19]
11. Polymethylmethacrylate	PMMA	114.5	0.33–0.34 [8, 20]
12. Polystyrene	PS	144.0	0.28 [8]
13. α-quartz, z-cut (0001)	Quartz	147.0	0.118 [7]
14. Silica, fused	Silica	155.5 [2]	0.184 [7]
15. Glass, silicate	Glass	158.5 [1]	0.225 [14]
16. Tile, glazed concrete	Tile	173.0	0.08–0.18 [21]
17. Cork	Cork	165.0	0.000 [21]

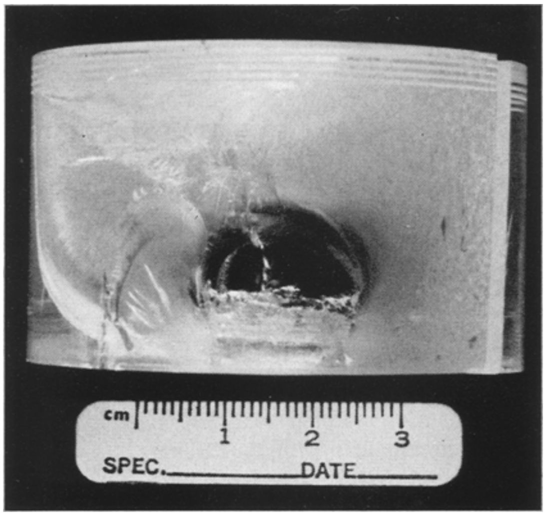

Figure 1 PMMA lens. Similar to PS.

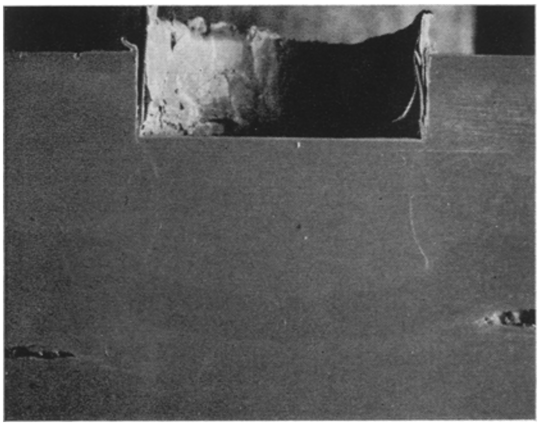

Figure 2 Cross-sectioned PVC lens. Similar to PPO.

cracks are very similar to those formed in concrete tile, cork and quartz. It should be noted here that the PU lenses relaxed back to their initial density and appearance within one month at room temperature but lenses of non-elastomers did not revert to their initial density even after three months.

A plot of the angle α against μ^2, in Fig. 5, indicates that there are two regions of linear relationship between α and μ^2. In the first region, encompassing all the organic polymers with Poisson's ratio in the range of 0.500 down to 0.265, the relationship follows the equation

$$\mu^2 = 0.250 - 1.194 \times 10^{-3}\alpha\,(\text{deg}); \qquad (1)$$
$$0^\circ \leqslant \alpha \leqslant 150^\circ\,.$$

In the second region, containing the various inorganic glasses, concrete, cork and quartz, with Poisson's ratio of 0.265 to 0.000, the relationship is

$$\mu^2 = 0.60 - 3.500 \times 10^{-3}\alpha\,(\text{deg}); \qquad (2)$$
$$150^\circ \leqslant \alpha \leqslant 170^\circ\,.$$

The reason for the extrapolation to $\alpha = 170^\circ$ when $\mu = 0.000$, instead of the intuitively expected $\alpha = 180^\circ$, is not yet understood.

A few lenses were extracted from the bodies of the samples; then the densities, *d*, of the

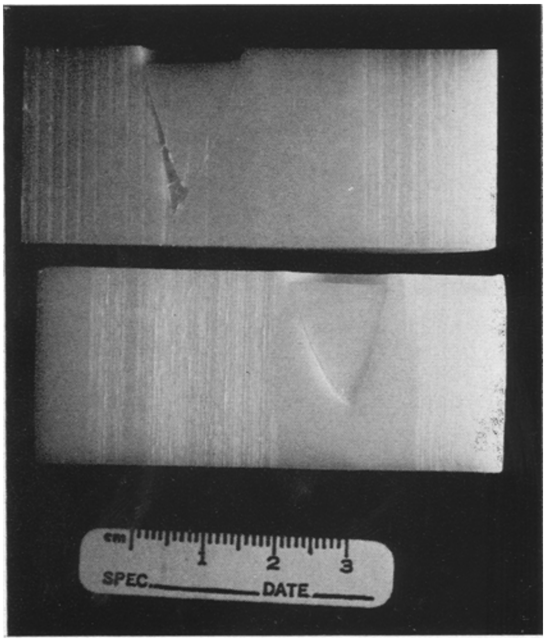

Figure 3 Cross-sectioned POM lens. Similar to nylon 66 and PP.

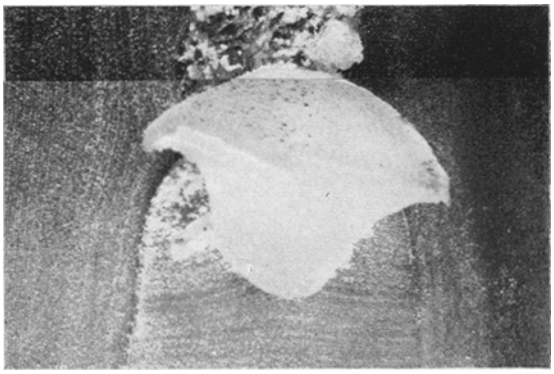

Figure 4 Cross-sectioned PU lens. Similar to NR. Lens was dislodged during sectioning.

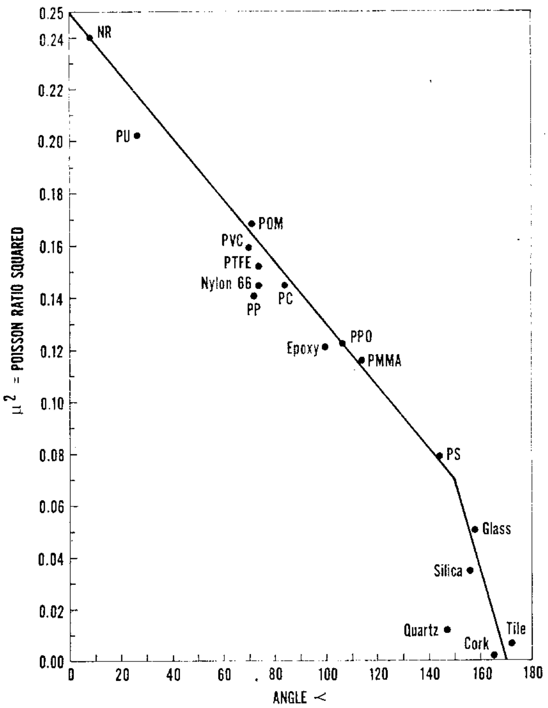

Figure 5 Relationship between the angle α and μ^2.

lenses and the corresponding bodies were determined at room temperature. The results are presented in Table II, where Δd is the density difference and $\Delta d/d_{\text{body}}$ is the density difference normalized with respect to the body density. The index of refraction, n, of PMMA sample impacted at 25°C was found to be $1.499 < n < 1.500$ for the lens and $n = 1.4980$ for the body of the sample. Applying the Lorenz-Lorentz relation between refractive index and density

$$\Delta d/d = \Delta n . 6n/(n^2 + 2)(n^2 - 1) \qquad (3)$$

where Δn is the change in the refractive index, one finds that $\Delta d/d = (2.5 \pm 0.5) \times 10^{-3}$, in excellent agreement with the actual density measurements. This densification is about ten times smaller than the one given by Puttick [5] and ten times larger than the one reported by Kimmel and Uhlmann [22] for the same material and temperature and about the same applied pressure.

Because of the decrease in magnitude of Young modulus E with an increase in temperature, below T_g, an increased densification of PMMA with temperature is expected [23]:

$$\Delta d/d = \Delta\sigma(1 - 2\mu)/E \qquad (4)$$

where $\Delta\sigma$ is the stress increment. The experimental results are in agreement with expectation. It is significant to notice that during the impact loading the ductile material flow within the lenses is associated with a densification of material and not with its dilation. A material flow under such conditions seems to be contradictory to the free volume theories of flow, according to which an enhanced flow is linked with an increase in free volume [23, 24]. These theories explain in a straightforward way the enhanced flow under tensile stress where dilation can occur, but fail to account for enhanced flow under compressive stress where contraction occurs. A Doolittle [24] type equation to correlate viscosity with free volume does not hold in this case since the flow occurs under densification which, according to the theory, should increase the viscosity and not decrease it. An enhanced flow under compressive stress was observed by this author during tests of wear of polymers [25] and in fracture of "brittle" polymers [26] when such tests were conducted under high compressive stresses.

4. Discussion

It was shown by Freudenthal [27] that for a uniformly distributed load, as it is in our case, the lines of constant maximum shearing stress, $\tau_{\max}$ = constant, are circles passing through the end-points of the loading surface. The circle of

TABLE II Densification under impact

No. and material	Impact temp. (°C)	Impact pressure (kbar)	Density of lens (g cm^{-3})	Density of body (g cm^{-3})	$\Delta d/d_{\text{body}}$
1. PP	25	5.0	0.8888	0.8873	1.69×10^{-3}
2. PMMA	−18	2.8	1.1745–1.1750	1.1730–1.1735	1.28×10^{-3}
3. PMMA	25	2.8	1.1785–1.1790	1.1755–1.1760	2.55×10^{-3}
4. PMMA	77	2.8	1.1565–1.1570	1.1515–1.1520	4.35×10^{-3}

highest possible value of $\tau_{\max}$ is a circle whose diameter equals the length of the loading surface. The pressure, $P_{\max}$, under which the first yielding occurs along this circle is obtained from the yield conditions. Under pressures $P > P_{\max}$ the plastic deformation spreads but is contained between adjacent circles of $\tau_{\max}$ = constant [27]. The resultant ductile volume resembles a lens and is so named.

Because the lens is confined by the loading surface and by the bulk of the loaded body, the stresses within it are compressive and no shearing can take place. If the value of $\tau_{\max}$ = constant on the surface of the lens exceeds the critical stress τ_c of the material, a crack will propagate through the interface between the lens and the body enveloping it. As was shown by Frank and Lawn [3], this crack will proceed orthogonally to the major normal stress σ_r, following a surface described by the trajectories of the other two normal stresses σ_z and σ_θ. Starting on the line separating the loaded from the unloaded surface, the σ_z trajectory carries the crack in a circle around the area of contact while the σ_θ trajectory carries the crack inward into the bulk of the material.

It is expected, therefore, that when $\tau_{\max}$ across the lens-body interface does not exceed τ_c, the lens will remain attached to the rest of the body with no cracks developing between them. When $\tau_{\max}$ exceeds τ_c at the borderline, cracks orthogonal to σ_r will form. In very brittle substances the crack will form cones, unable to change direction once propagation started [3]. In less brittle materials the cracks will be able to retain orthogonality to σ_r, curve during propagation, and envelop completely or partially the lenses. Our experimental results fully corroborate these expectations.

The dependence of the lens's shape on Poisson's ratio was discussed by Hunt [6] and by Hartley and Wilshaw [7]. They dealt with substances having relatively small μ. Hunt calculated a weak dependence of the location of the lens's centre on Poisson's ratio. Hartley and Wilshaw showed that a relationship between μ and the diameter of a circular crack formed under Hertzian conditions exists, but due to the anisotropy of their α-quartz they did not generalize any conclusion.

The experimental data indicates unequivocally that there exists a correlation between the shape of the lens and the Poisson ratio of the respective material. This correlation is manifested in two linear relationships between α and μ^2 where α is the angle between the loaded surface and the shoulder of the lens. The fact that there are two regions of linearity seems to indicate that the relationship does not depend solely on μ and that the linear α-μ^2 relationships actually reflect a correlation between the shape of the lens and basic properties such as material moduli. This point, however, requires further study.

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